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Fluctuations in Ground Temperature with Variable Suction Velocity and a Convective Boundary Condition

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Abstract- This paper considers a convective flow of heat into the ground through the surface with a variable suction velocity. The problem is two dimensional with the soil surface taken to be optically thin environment which makes the radiative heat transfer significant. All other soil properties are assumed to be constant. The governing dimensional equations were reduced to non-dimensional form using some dimensionless parameters, and the transient non-linear partial differential equation (PDE) was solved analytically. Asymptotic method was adopted to linearize the PDE and the resulting homogeneous and non-homogeneous ordinary differential equations were solved using undetermined coefficients methods. The results were shown graphically. It is observed that temperature fluctuates sinusoidally with time.

Keywords: Asymptotic method, Biot number, Fluctuation, Ground Temperature, Prandtl number, Radiation parameter, Suction Velocity.

1. Introduction:

The subject of ground temperature is a very important one which has been gaining more attention as the day goes by. This is because the ground with its properties plays so much significant roles in various fields of study. It ranges from agriculture to constructions in civil engineering, architecture and building technology. Because of this wide range of use of the ground, many researchers have developed interest into exploring the behaviour of the ground under different situations. For example, in drying process, energy transfer in a wet cooling tower, the process of evaporation at the surface of water body, etc, heat and mass transfer occur simultaneously. Achievable use of this type of flow may be seen in many industries. In power industry for example, among methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. This is of a huge practical importance to engineers and scientists because of its frequent occurrence in many branches of science and engineering, [1].

Exact measurement of ground heat flux is a difficult task. According to Oke [2], heat flux plates can be used to measurement soil heat flux directly. However, the flux sensor usually needs to be positioned normal at a few centimetres below the surface at a certain depth in the soil to avoid disturbances. Relatively fast movement of water (both liquid and vapour) near the surface of the soil affects the precision of the measurements by heat flux disks owing to poor thermal contact between the plate and soil [3].

Makinde and Bello [4] studied the Effects of Soil Temperature Pattern on the Performance of Cucumber Intercrop with Maize in a Tropical Wet-and-Dry Climate of Nigeria. They discovered that mean soil temperature of 33°C and 31°C at 5 and 10cm during late season could be said to have enhanced the productivity of cucumber yield by about 50% compared to early season with mean soil temperature of 30°C and 29°C at 5cm and 10cm below soil surface. The mean cucumber yield of 9t/ha and 6.1t/ha for mono and mixed crop respectively during early season trial was significantly lower ($P<0.05$) than the mean cucumber yield of 15.34t/ha and 12.34t/ha for late season. Akinpelu, Olaleye and Adewoye [5] investigated the effects of some physical parameters on ground temperature with time-dependent suction velocity in the presence of internal heat generation. Considering a great depth into the ground where the internal heat generation is much more significant, their results showed that the increase in prandtl number and radiation parameter increased the temperature. However, the temperature fluctuates with time. Okedoye [6] investigated the analytical analysis of steady Magneto-hydrodynamics (MHD) free convective heat and mass transfer flow past a semi-infinite vertical porous plate in a porous medium. His results showed that temperature profile decreased with increase in Prandtl, but increased with increasing Dufor number. Akinpelu, Alabison and Olaleye [7] studied the variations in ground temperature in the presence of radiative heat flux and spatial dependent soil thermophysical property. They observed that the increase in prandtl number and radiation parameter decreased the temperature. The temperature also fluctuates with time at different values of the frequency of oscillation and depth. The investigations of Nwaigwe [8] also revealed decrease in ground temperature at increasing prandtl number and radiation parameter. The author however imposed a Dirichlet type of boundary condition. Sharma [9] studied the unsteady free convective viscous incompressible flow past an infinite vertical porous flat plate with periodic heat and mass transfer in slip-flow regime. He discovered that the temperature increased near the plate but decreased far away from the plate.

In this research work, effort was however made to consider the ground temperature involving convective boundary conditions with time dependent suction velocity.

2. Formulation of the Problem

In this research work, we assumed that the heat is flowing into the ground through the surface, and this is maintained at a constant temperature, $T^{'}$. The flow is considered in the vertical direction ($z^{'}$ - axis) only. It is also assumed that the surface of the soil is an optically thin environment and that the radiative heat transfer is significant. The thermal conductivity k is considered to be constant, i.e. $k = k_0$. Other properties like specific heat capacity, C_p and density, ρ are also constant. The surface of the soil is horizontally homogeneous with vegetative cover.

Under the above assumptions and before the introduction of dimensionless variables, the transient heat transfer equations of the flow are as follows:

$$\frac{\partial w'}{\partial z'} = 0 \tag{1}$$

And

$$\frac{\partial T^{'}}{\partial t^{'}} + w^{'} \frac{\partial T^{'}}{\partial z^{'}} = \frac{1}{\rho C_p} \frac{\partial}{\partial z^{'}} \left(k \frac{\partial T^{'}}{\partial z^{'}} \right) - \frac{1}{\rho C_p} \frac{\partial q_r^{'}}{\partial z^{'}} \tag{2}$$

The boundary conditions considered in this work are:

$$k^* \frac{\partial T'(z,t)}{\partial z'} = h(T_w' - T_\infty) \; on \; z = 0 \qquad \text{(Wall temperature)}$$

$$T'(z,t) = T_\infty \; as \; z \to \infty \qquad \text{(Temperature at infinite depth)} \tag{3}$$

Where,

$k^* = coefficient \; of \; thermal \; conductivity$

$h = coefficient \; of \; convection \; heat \; transfer$

Equation (3) is described as the Convective boundary condition.

Since the soil is an optically thin environment, radiative flux takes the form [6]:

$$\frac{\partial q'}{\partial z'} = 4\alpha^2 (T' - T_\infty) \tag{4}$$

Where, $\alpha =$ The absorption coefficient

Equation (2) becomes

$$\frac{\partial T^{'}}{\partial t^{'}} + w^{'} \frac{\partial T^{'}}{\partial z^{'}} = \frac{1}{\rho C_p} \frac{\partial}{\partial z^{'}} \left(k \frac{\partial T^{'}}{\partial z^{'}} \right) - \frac{4\alpha^2}{\rho C_p} \left(T^{'} - T_\infty \right) \tag{5}$$

From equation (1), w' is not a function of z'.It is either a constant or a function of time.

Thus,

$w' = -w_0' \; or \; w' = -w_0'(1 + \varepsilon A e^{i\omega' t})$

Where,

w' *is the initial suction velocity*.

For the purpose of this work w' is taken to be $w' = -w_0'(1+\varepsilon A e^{i\omega' t})$ (6)

The negative sign indicates that suction is towards the ground surface. ω *is* the frequency of oscillation. A and ε are very small positive parameters such that their product is far less than one, that is $\varepsilon A << 1$

Substituting equation (6) into equation (5), we have

$$\frac{\partial T'}{\partial t'} - w_0'\left(1+\varepsilon A e^{i\omega' t}\right)\frac{\partial T'}{\partial z'} = \frac{1}{\rho C_p}\frac{\partial}{\partial z'}\left(k\frac{\partial T'}{\partial z'}\right) - \frac{4\alpha^2}{\rho C_p}\left(T' - T_\infty\right) \quad (7)$$

Introducing the following dimensionless parameters:

$$t = \frac{w_0'^2 t'}{4\vartheta},\ z = \frac{w_0' z'}{\vartheta} = \frac{z'}{L},\ \omega = \frac{4\vartheta\omega'}{w_0'^2},\ w = \frac{w'}{w_0'},\ \theta = \frac{T' - T_\infty}{T_w' - T_\infty},\ \vartheta = \frac{\mu}{\rho},\ P_r = \frac{\mu C_p}{k},$$

$$R^2 = \frac{4\vartheta\alpha^2}{\rho C_p w_0'^2} \quad (8)$$

Equation (7) simplifies to:

$$\frac{1}{4}P_r\frac{\partial\theta}{\partial t} - P_r\left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial\theta}{\partial z} = \frac{1}{k_0}\frac{\partial}{\partial z}\left(k\frac{\partial\theta}{\partial z}\right) - R^2\theta \quad (9)$$

Equation (3) also simplifies to:

$\frac{\partial\theta(z,t)}{\partial z} = B_i$ *on* z = 0 (wall temperature)

$\theta(z,t) \to 0$ *as* $z \to \infty$ (Temperature of the soil profile at infinite depth) (10)

Where, B_i = Biot number

Equation (9) therefore is the governing equation, subject to equation (10).

Where, $R^2 = \frac{4\alpha^2\vartheta^2}{k_0 w_0'^2}$ (R is the radiative parameter), $P_r = \frac{\mu C_\rho}{k_0}$ is the Prandtl number,

$A =$ suction velocity parameter, $k = k_0$ is the constant thermal conductivity of the soil.

3. Method of Solution

Substitute $k = k_0$ into equation (9) to get

$$\frac{1}{4}p_r\frac{\partial\theta}{\partial t} - p_r(1+A\varepsilon e^{i\omega^1 t})\frac{\partial\theta}{\partial z} = \frac{\partial^2\theta}{\partial z^2} - R^2\theta \quad (11)$$

Subject to

$$\frac{\partial\theta}{\partial z} = B_i \ on\ z = 0\ and\ \theta \to 0\ as\ z \to \infty$$

Where,

$P_r = \frac{\mu C_\rho}{k_0}$, $R^2 = \frac{4\alpha^2 \vartheta^2}{k_0 w_0'^2}$ and B_i = Biot number

Equation (11) is a partial differential equation and since ε is small, a time dependent perturbation method can be adopted. Solution of the following form is assumed:

$$\theta(z,t) = \theta_0(z) + \theta_1(z)\varepsilon e^{i\omega t} \tag{12}$$

Substituting equation (12) into equation (11) and compiling the orders of ε and their corresponding boundary conditions:

Order ε^0

$$\frac{d^2\theta_0}{dz^2} + P_r \frac{d\theta_0}{dz} - R^2\theta_0 = 0 \tag{13}$$

Subject to

$$\frac{d\theta_0}{dz} = B_i \; on \; z = 0 \; and \; \theta_0 \to 0 \; as \; z \to \infty \tag{14}$$

Order ε^1

$$\frac{d^2\theta_1}{dz^2} + P_r \frac{d\theta_1}{dz} - (\frac{i\omega}{4} P_r + R^2)\theta_1 = -P_r A \frac{d\theta_0}{dz} \tag{15}$$

Subject to

$$\frac{d\theta_1}{dz} = 0 \; on \; z = 0 \; and \; \theta_1 \to 0 \; as \; z \to \infty \tag{16}$$

Solving equations (13) and (15), and applying their boundary conditions give

$$\theta_0(z) = \gamma_1 B_i e^{m_2 z} \tag{17}$$

$$\theta_1(z) = N_{20} G_1 \tag{18}$$

Based on equations (17) and (18), equation (12) becomes

$$\theta(z,t) = \gamma_1 B_i e^{m_2 z} + N_{20} G_1 \varepsilon e^{i\omega t} \tag{19}$$

Hence, the required solution.

Where,

$N_{20} = \frac{\gamma_1 \gamma_2 B_i P_r A m_2}{Q_{00}}$, $\gamma_1 = -\frac{1}{m_2}$ $\gamma_2 = -\frac{m_2}{m_4}$, $B_i = Biot \; number$

$G_1 = e^{m_2 z} + \gamma_2 e^{m_4 z}$, $m_2 = -\frac{1}{2}\left\{P_r + \sqrt{Q_1}\right\}$ $m_4 = -\frac{1}{2}\left\{P_r + \sqrt{Q_7}\right\}$,

$Q_{00} = \frac{i\omega}{4} P_r$ $Q_1 = P_r^2 + 4R^2$, $Q_7 = Q_1 + 4Q_{00}$

4. Discussion of Results

The approximate analytical solution is graphically displayed to illustrate the effect of the various physical parameters of interest on the model.

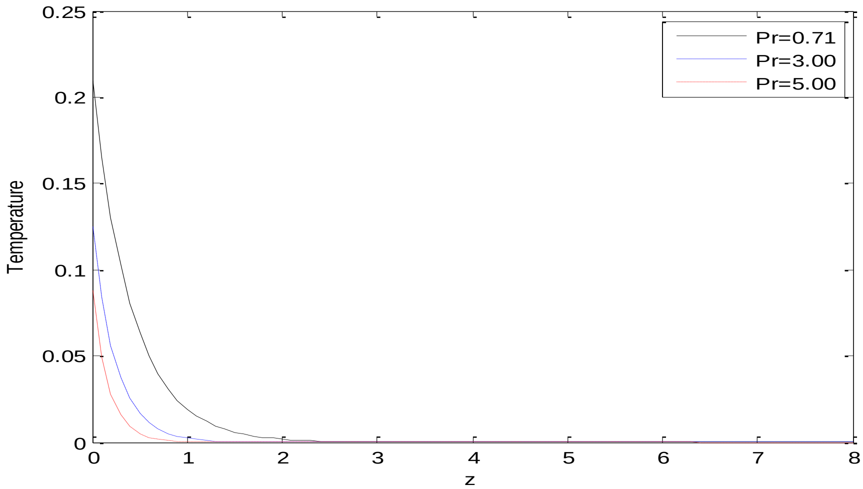

Figure 1: Temperature profiles for $\varepsilon = 0.01, \omega = \pi/2, t = 1, A = 1, R = 2, B_i = 0.5$ at different values of P_r

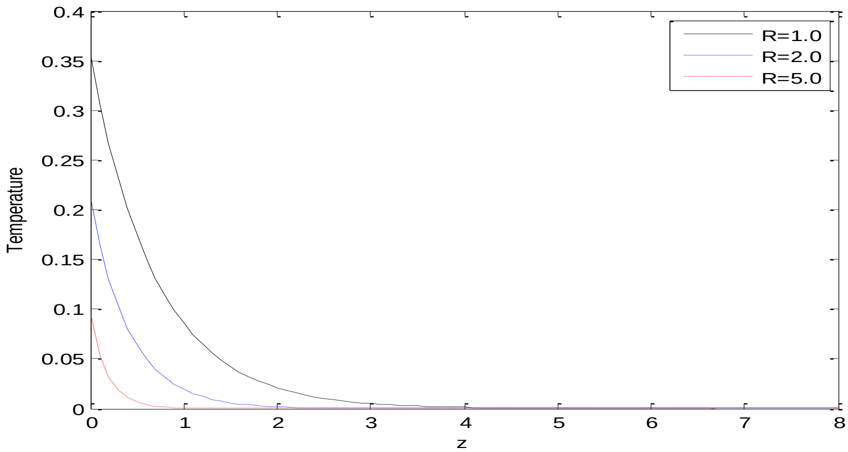

Figure 2: Temperature profiles for $\varepsilon = 0.01, \omega = \pi/2, t = 1, A = 1, P_r = 0.71, B_i = 0.5$ at different values of R

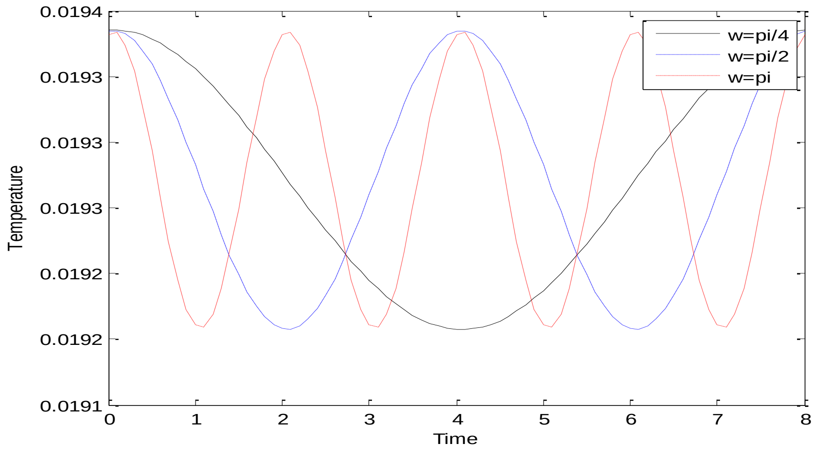

Figure 3: Temperature profiles for $\varepsilon = 0.01, R = 2, A = 1, P_r = 0.71, B_i = 0.5$ at different values of ω

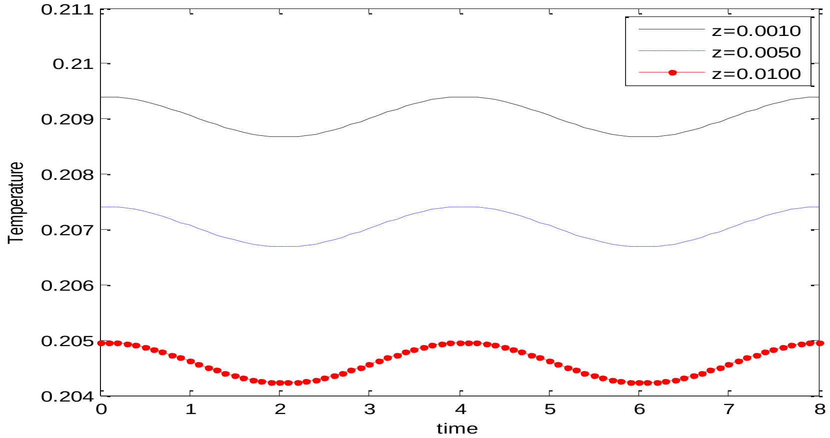

Figure 4: Temperature profiles for $\varepsilon = 0.01, R = 2, A = 1, \omega = \pi/2, P_r = 0.71, B_i = 0.5$ at different values of depth (z)

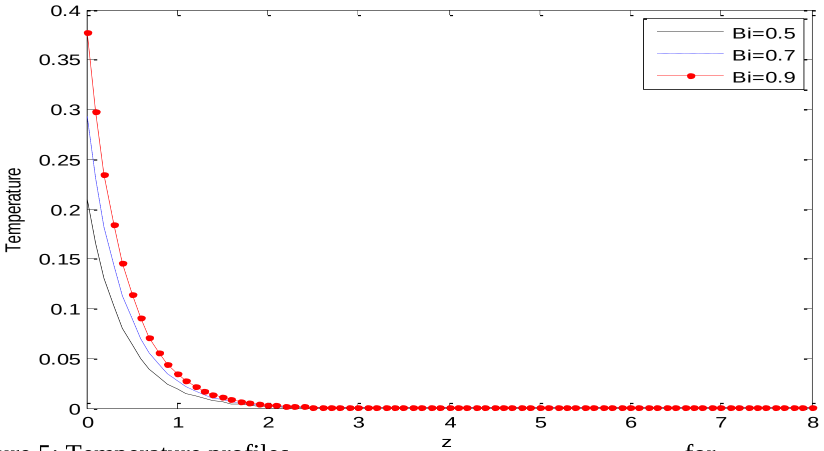

Figure 5: Temperature profiles for $\varepsilon = 0.01, R = 2, A = 1, \omega = \pi/2, t = 1, P_r = 0.71$ at different values of biot number (B_i)

Tables 1 – 2: Effect of the other parameters on the temperature profile.

Table 1

z=0	epsi=0.01	epsi=0.05	epsi=0.10
0.5	0.0807	0.0808	0.0809
1	0.0245	0.0245	0.0246
1.5	0.0074	0.0075	0.0075
2	0.0023	0.0023	0.0023
2.5	0.0007	0.0007	0.0007
3	0.0002	0.0002	0.0002
3.5	0.0001	0.0001	0.0001
4	0	0	0
4.5	0	0	0

Table 2

z=0	A=1.0	A=5.0	A=10.0
0.5	0.0807	0.0808	0.0809
1	0.0245	0.0245	0.0246
1.5	0.0074	0.0075	0.0075
2	0.0023	0.0023	0.0023
2.5	0.0007	0.0007	0.0007
3	0.0002	0.0002	0.0002
3.5	0.0001	0.0001	0.0001
4	0	0	0
4.5	0	0	0

Figures 1 – 5 show the behaviour of the ground temperature for different values of the flow parameters. Figure 1, 2 and 5 portray the temperature profile at various prandtl number (P_r), Radiation parameter (R) and Biot number respectively. As these all increase, the temperature decreases with corresponding increasing depth. Figures 3 and 4 show the sinusoidal variation of temperature of the soil with time at each frequency of oscillation of the suction velocity and each depth of the soil respectively. The former reveals that the period of the variation of the temperature of the ground decreases with increase in the frequency of oscillation of the suction velocity. The latter shows that the amplitude of variation of the soil temperature decreases with depth and becomes insignificant as the depth increase.

Tables 1 – 2 illustrate the temperature profile for different values of the flow variables that are not considered in figures 1 – 5. It can be seen from the tables that the temperature of the ground decreases with increase in the depth. It can also be noticed that increase in the asymptotic parameter, ε and the suction velocity parameter, A decreases the temperature profile respectively. This effect becomes insignificant with increase in depth.

5. Conclusions

This study investigated the fluctuations in ground temperature under a convective boundary condition and variable suction velocity. An approximate analytical solution is obtained for the problem. The effects of some physical parameters of interest on ground temperature were also examined. It was discovered that the ground temperature fluctuates with time but decreased with increasing prandtl number, Radiation parameter and biot number as the depth deepens.

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